

• $\int_{-\infty}^{\infty} \delta(x) dx = 1$ (normalization)
• $\int_{-\infty}^{\infty} x \delta(x) dx = 0$ (centered at 0)
• $\int_{-\infty}^{\infty} x^n \delta(x) dx = 0$ for $n > 0$

• $\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = f(a)$
• $\int_{-\infty}^{\infty} f(x) \delta(x) dx = f(0)$

Dirac Delta Function

• $\delta(x)$ is a distribution, not a function.
• $\delta(x)$ is defined by its action on a test function $f(x)$.

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• $\delta(x)$ is symmetric:

$$\delta(x) = \delta(-x)$$

$$\int_{-\infty}^{\infty} f(x) \delta(x-a) dx = \int_{-\infty}^{\infty} f(-x) \delta(-x-a) dx$$

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• $\delta(x)$ is a limit of a sequence of functions.

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